Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 14 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01R**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 6 3 8 9 A 0 1 2 8

Pearson

1. The sequence $u_1, u_2, u_3, \ldots$ satisfies

$$\boxed{u_{n+2} = 3u_{n+1} - 2u_n}$$

recursive sequence (plugging in a term on the right gives the next term on the left)

Given that

- $u_1 = 7$ starting point
- $u_3 = 4$

(a) find the value of u_2 **(2)**

(b) find $\sum_{r=1}^{4} (u_r + 2r)$ **(3)**

end, terms, sum, starting value

(a) This is a recursive sequence:

$$u_{n+2} = 3u_{n+1} - 2u_n$$

↳ notice how:

$n+2$ is 2 more than n

$n+1$ is 1 more than n

$n+2$ is 1 more than $n+1$

$$u_3 = 3u_2 - 2u_1$$

$$4 = 3u_2 - 2(7)$$

$$3u_2 = 18 \rightarrow u_2 = 6$$

(b) $\sum_{r=1}^{4} (u_r + 2r)$ means start the sequence at $r=1$ and sum all the terms up to $r=4$

① Find u_4:

$$u_4 = 3u_3 - 2u_2 = 3(4) - 2(6) = 0$$

Question 1 continued

② substitute into sum :

remember : $u_1 = 7$ $u_3 = 4$
$u_2 = 6$ $u_4 = 0$

$$\sum_{r=1}^{4}(u_r+2r) = (u_1+2(1)) + (u_2+2(2)) + (u_3+2(3)) + (u_4+2(4))$$

$$= (7+2(1)) + (6+2(2)) + (4+2(3)) + (0+2(4))$$

$$= 37$$

(Total for Question 1 is 5 marks)

2. The table shows corresponding values of x and y for a continuous curve with equation $y = \mathrm{f}(x)$ between $x = -4$ and $x = 5$, where a is a constant.

Interval btwn two x-coords = h of trapezium

x	-4	-2.5	-1	0.5	2	3.5	5
y	4.16	2.91	a	1.73	1.37	1.43	2.28

y_7

y_1 multiplied by 2 as all are shared by two trapezia

The trapezium rule is used with all the y values in the table to find an approximation for

$$\int_{-4}^{5} \mathrm{f}(x)\,\mathrm{d}x$$

Given that the value of this approximation is 19.3

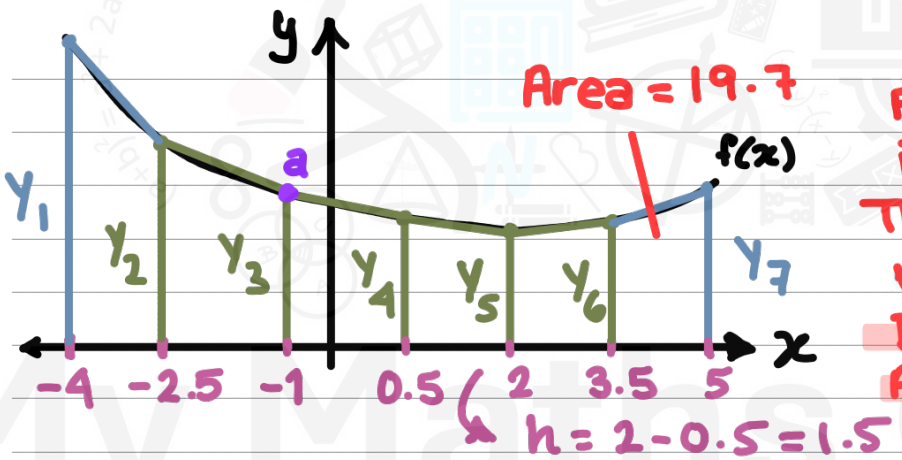
(a) find the value of the constant a to 3 significant figures.

(3)

(b) Use the given answer of 19.3 to find an approximate value for

$$\int_{-4}^{5} (2\mathrm{f}(x) - 3)\,\mathrm{d}x$$

(2)



Formula is given in formula booklet. The graph is only a visual representation. DO NOT DRAW A GRAPH IN THE EXAM

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4 + y_5 + y_6) + y_7\right]$$

$$19.3 = \frac{1}{2}(1.5)\left(4.16 + 2(2.91 + a + 1.73 + 1.37 + 1.43) + 2.28\right)$$

$$19.3 = \frac{3}{4}\left(6.44 + 2(a + 7.44)\right)$$

Question 2 continued

$19.3 = \frac{3}{4}(6.44 + 2a + 14.88)$

$19.3 = \frac{3}{4}(21.32 + 2a)$

$19.3 = 15.99 + 1.5a \longrightarrow a = 2.207$

$a = 2.21$ (3 s.f.)

(b) Use rules:

1. $\int af(x)\,dx = a\int f(x)\,dx$
2. $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$
3. $\int ax^n\,dx = \frac{a}{n+1}x^{n+1}$

$\int_{-4}^{5} 2f(x) - 3\,dx$

$= 2\int_{-4}^{5} f(x)\,dx - \int_{-4}^{5} 3^0\,dx$

$= 2(19.3) - 3x\Big]_{-4}^{5}$

$= 38.6 - (3(5) - 3(-4)) = 11.6$

(Total for Question 2 is 5 marks)

3. A circle has equation

$$x^2 + y^2 + 8x - 14y - 79 = 0$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

point (0,0)

(3)

Given that P is the point on the circle that is nearest the origin O,

(b) find the exact length of OP

(2)

(a) Standard form of equation of a circle:

$$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$$

↳ rewrite given eq. into standard form:

$$x^2 + 8x + y^2 - 14y - 79 = 0$$

(1) complete the square for terms with x:

$$x^2 + 8x + y^2 - 14y = 79$$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides:

$$x + 8x + \left(\frac{8}{2}\right)^2 + y^2 - 14y = 79 + \left(\frac{8}{2}\right)^2$$

$$x + 8x + 16 + y^2 - 14y = 79 + 16$$

$$x + 8x + 16 + y^2 - 14y = 95$$

↳ factorise

$$(x+4)^2 + y^2 - 14y = 95$$

(3) complete the square for terms with y:

$$(x+4)^2 + y^2 - 14y = 95$$

Question 3 continued

↳ add $\left(\frac{b}{2}\right)^2$ to both sides :

$$(x+4)^2 + y^2 - 14y + \left(\frac{14}{2}\right)^2 = 95 + \left(\frac{14}{2}\right)^2$$

$$(x+4)^2 + y^2 - 14y + 49 = 95 + 49$$

$$(x+4)^2 + y^2 - 14y + 49 = 144$$

↳ factorise

$$(x+4)^2 + (y-7)^2 = 144$$

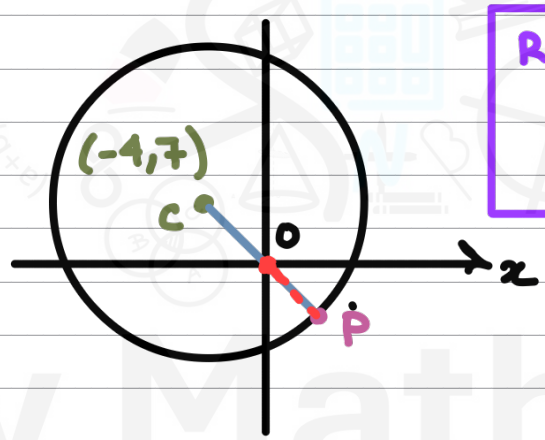
$-(-)$

so : $(x+4)^2 + (y-7)^2 = 144$

(i) coords of centre $= (-4, 7)$

(ii) radius $= \sqrt{144} = 12$

(b)



RECALL : distance formula :

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

length OP = r − distance CO

$$= 12 - \sqrt{(-4-0)^2 + (7-0)^2}$$

$$= 12 - \sqrt{16+49}$$

$$= 12 - \sqrt{65}$$

(Total for Question 3 is 5 marks)

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3+2x)^6 = (a+bx)^n$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{6x}\right)(3+2x)^6$$

(3)

(a) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) − 1

① Find coefficients :

${}^6C_0 = 1 \longrightarrow$ (FIRST AND LAST ${}^nC_r$ ARE ALWAYS = 1).

$${}^6C_1 = \frac{6!}{(6-1)!\,1!} = 6$$

$${}^6C_2 = \frac{6!}{(6-2)!\,2!} = 15$$

$${}^6C_3 = \frac{6!}{(6-3)!\,3!} = 20$$

② expand :

$$(3+2x)^6 = 1(3)^6(2x)^0 + 6(3)^{6-1}(2x)^1 + 15(3)^{6-2}(2x)^2 + 20(3)^{6-3}(2x)^3$$

$$= 729 + 2916x + 4860x^2 + 4320x^3 + \dots$$

Question 4 continued

(b) $\left(2x^2 - \frac{1}{6x}\right)(3+2x)^6$

We can multiply each term in $(2x^2 - 1/6x)$ by each term of the expansion of $(3+2x)^6$:

$$= \left(2x^2 - \frac{1}{6x}\right)\left(729 + 2916x + 4860x^2 + 4320x^3 + \dots\right)$$

- Find all the values that multiply to give a term with x^2:

$2x^2 \times 729 = 1458x^2$

$\frac{-1}{6\cancel{x}} \times 4320x^{\cancel{3}\,2} = -720x^2$

- Then, add the terms:

$1458x^2 + (-720)x^2 = 738x^2$

coefficient $= 738$

COMMON MISTAKE: DO NOT ADD $4860x^2$ TO THE TERMS, SINCE IT NO LONGER HAS VARIABLE x^2 WHEN IT IS MULTIPLIED BY $2x^2$ OR $\frac{1}{6x}$

(Total for Question 4 is 7 marks)

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

12 PM

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

(a) midnight = 12 AM

2 AM = 2 hours after midnight

So $t = 2$

$D = 8 + 5\sin\left(\frac{\pi(2)}{6} + 3\right)$

$= 8 + 5\sin\left(\frac{\pi}{3} + 3\right)$

DON'T FORGET TO SET CALCULATOR TO RADIANS!

$D = 4.07$ (3 s.f.)

$4.07 > 4$

(b) midday = 12 PM

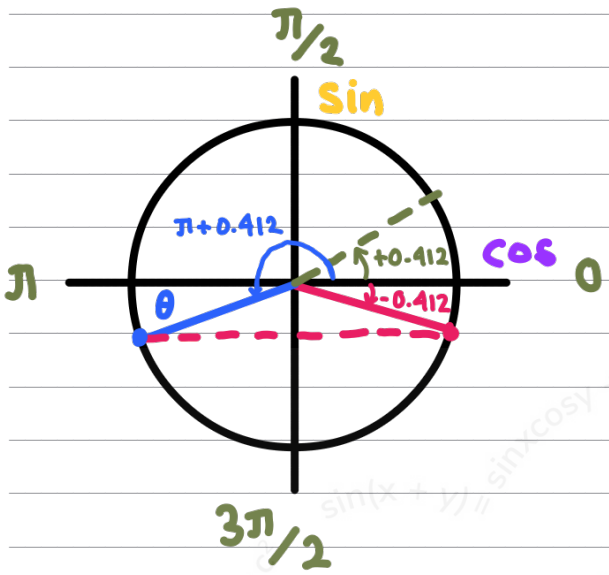
$\left[6 = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right)\right] \div 5$

$\sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5}$

$\frac{\pi t}{6} + 3 = \arcsin\left(-\frac{2}{5}\right)$

Question 5 continued

ON UNIT CIRCLE :



→ calculator angle for $\arcsin\left(-\frac{2}{5}\right) = -0.4115...$

We need a +ve value for t, so use :

$\theta = \pi + 0.4115$

$$\frac{\pi t}{6} + 3 = \pi + 0.4115$$

$$\frac{\pi t}{6} + 3 = 3.55$$

$$t = \frac{6(3.55-3)}{\pi} = 1.056 \text{ hours}$$

$1.056 \times 60 = 63.38$ mins

Since the question is asking for the first time, we must round UP; nearest minute

t:	63	63.38	64
d:	<6	=6	>6

So t = 64 minutes after midday = 1:04 PM OR 13:04

(Total for Question 5 is 6 marks)

6. $$f(x) = 4x^3 + px^2 + 8x + q$$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

two variables implies we will need to form simultaneous equations.

gradient < 0

(a) (i) ① set the factor equal to zero :

$2x+3=0$

$x = -\frac{3}{2}$

② substitute $x = -\frac{3}{2}$ into $f(x) = 0$:

$4x^3 + px^2 + 8x + q = 0$

$4\left(-\frac{3}{2}\right)^3 + p\left(-\frac{3}{2}\right)^2 + 8\left(-\frac{3}{2}\right) + q = 0$

$-\frac{27}{2} + \frac{9}{4}p - 12 + q = 0$

$\boxed{\frac{9}{4}p + q - \frac{51}{2} = 0}$ → EQUATION 1

③ USE :

REMAINDER THEOREM :

if $\frac{f(x)}{(x-k)}$ = quotient + remainder :

then $f(k)$ = remainder

so : $(x+2)$ gives remainder -5

therefore $f(-2) = -5$

Question 6 continued

④ Substitute $x = 2$ to $f(x) = -5$:

$$4x^3 + px^2 + 8x + q = -5$$
$$4(-2)^3 + p(-2)^2 + 8(-2) + q = -5$$
$$-32 + 4p - 16 + q = -5$$
$$\boxed{4p + q - 43 = 0} \rightarrow \text{EQUATION 2}$$

↳ SOLVE SIMULTANEOUS EQUATIONS:

METHOD 1: elimination:

$$4p + q - 43 = 0$$
$$\frac{9}{4}p + q - \frac{51}{2} = 0$$
$$4p + \cancel{q} = 43$$
$$\frac{9}{4}p + \cancel{q} = 51/2 \quad \ominus$$
$$\left(4 - \frac{9}{4}\right)p = 43 - \frac{51}{2}$$
$$\left[\frac{7}{4}p = \frac{35}{2}\right] \times \frac{4}{7}$$
$$p = 10$$

METHOD 2: substitution:

$$4p + q - 43 = 0$$
$$\frac{9}{4}p + q - \frac{51}{2} = 0$$
$$q = 43 - 4p$$
$$\frac{9}{4}p + 43 - 4p - \frac{51}{2} = 0$$
$$-\frac{7}{4}p + \frac{35}{2} = 0$$
$$\left[-\frac{7}{4}p = -\frac{35}{2}\right] \times -\frac{4}{7}$$
$$p = 10$$

(ii) Substitute $p = 10$ into any of the equations:

$$4(10) + q = 43$$
$$40 + q = 43$$
$$q = 3$$

Question 6 continued

(b) $f'(x) < 0$ when gradient is decreasing

$f(x) = 4x^3 + 10x^2 + 8x + 3$

$f'(x) = 3(4)x^2 + 2(10)x + 8$

$= 12x^2 + 20x + 8$

DIFFERENTIATION

$f(x) = ax^n$

$f'(x) = nax^{n-1}$

Find roots:

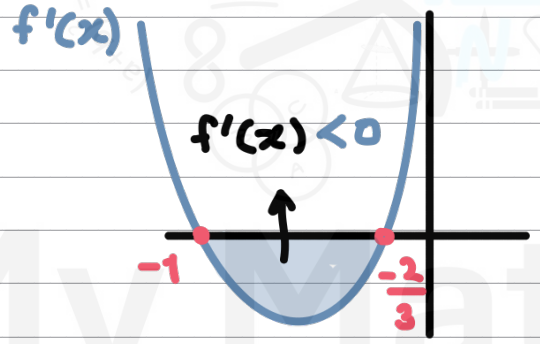
$12x^2 + 20x + 8 = 0$

$(12x + 8)(x + 1) = 0$

$4(3x+2)(x+1) = 0$

$x = -\frac{2}{3}$ or $x = -1$

IMAGINE THE GRAPH OF $f'(x)$; quadratic:



so gradient is decreasing when:

$-1 < x < -\frac{2}{3}$

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$, the equation

$$3\sin x \tan x = 11 + \cos x$$

giving the answers in radians to 3 decimal places.

(5)

(ii) Given that

- $0 < \theta < 90°$
- $\cos\theta = \frac{1}{3}$

find, in simplest form, the exact value of $\tan\theta$

(2)

(i) $3\sin x \tan x = 11 + \cos x \longrightarrow \tan x = \frac{\sin x}{\cos x}$

$3\sin x \frac{\sin x}{\cos x} = 11 + \cos x$

$\cos x\left(\frac{3\sin^2 x}{\cos x}\right) = (11 + \cos x)\cos x$

$3\sin^2 x = 11\cos x + \cos^2 x$

Use identity : $\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

$3(1 - \cos^2 x) = 11\cos x + \cos^2 x$

$\cancel{3 - 3\cos^2 x} = 11\cos x + \cos^2 x$

$\cancel{-3 + 3\cos^2 x} \quad -3 + 3\cos^2 x$

$4\cos^2 x + 11\cos x - 3 = 0$

↳ FACTORISE :

$(4\cos x - 1)(\cos x + 3) = 0$

$\cos x = 1/4$ or $\cos x = 3$ → REJECT ; range of $y = \cos x$ is $-1 \leq y \leq 1$

↳ SOLVE FOR x :

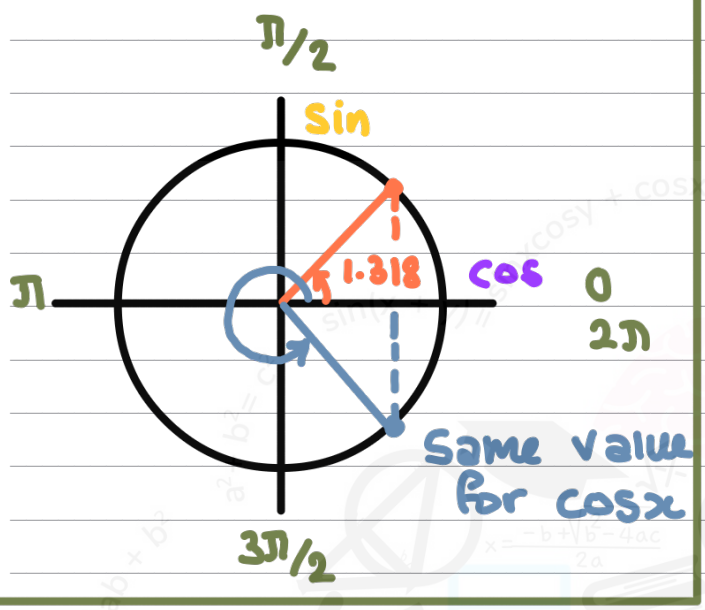
$\cos x = \frac{1}{4}$, $0 \leqslant x < 2\pi$

Question 7 continued

$x = \arccos \frac{1}{4}$

$x = 1.318$ (3 d.p.)

ON UNIT CIRCLE:



so: $x = 2\pi - 1.318$
$= 4.965$ (3 d.p.)

$x = 1.318, 4.965$

FINAL ANSWER!

(b) $\cos\theta = 1/3$, $0 < \theta < 90°$

Use identity: $\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = \frac{1}{3^2}$

$1 - \sin^2\theta = \frac{1}{9}$

$\sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$

$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}}{3} \div \frac{1}{3} = 2\sqrt{2}$

(Total for Question 7 is 7 marks)

8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

has common ratio (r).

$$k+4 \qquad k-2 \qquad 11-k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent, approaches finite value

(b) find the value of S_∞

(4)

(i) (a) In an arithmetic sequence, we add d to each term to find the next term.

So: $S_n = a + a+d + \ldots + a+(n-2)d + a+(n-1)d$

If we write the sum backwards:

$S_n = a+(n-1)d + a+(n-2)d + \ldots + a+d + a$

Now, we can add both to find double S_n:

$$\oplus \quad S_n = a + a+d + \ldots + a+(n-2)d + a+(n-1)d$$

$$S_n = a+(n-1)d + a+(n-2)d + \ldots + a+d + a$$

$$2S_n = 2a+(n-1)d + 2a+(n-1)d \quad 2a+(n-1)d + \underbrace{2a+(n-1)d}_{\text{term } a_n}$$

$$2S_n = n(2a+(n-1)d)$$

Question 8 continued

$\therefore S_n = \frac{n}{2}(2a+(n-1)d)$

(b) $900 + 892 + 884 + \ldots + 500$ term a_n

d

$a = 900$

$d = 892 - 900 = -8$

$a_n = a + (n-1)d$

$500 = 900 + (n-1)(-8)$

$n = \frac{500-900}{8} + 1 = 51$

$S_{51} = \frac{51}{2}(2(900) + (51-1)(-8)) = 35\,700$

(ii) (a)

a_1 $k+4$ $\quad$ a_2 $k-2$ $\quad$ a_3 $11-k$

r $\quad$ r

$\boxed{r = \frac{a_2}{a_1} = \frac{k-2}{k+4}}$ $\quad$ $\boxed{r = \frac{a_3}{a_2} = \frac{11-k}{k-2}}$

$\frac{a_2}{a_1} = \frac{a_3}{a_2}$ (r = r)

$\frac{k-2}{k+4} = \frac{11-k}{k-2}$ (cross multiply)

quick method

$(a+b)^2 = a^2+2ab+b^2$

$(k-2)^2 = (k+4)(11-k)$

$k^2 - \cancel{4k} + 4 = 11k - k^2 + 44 - \cancel{4k}$

$2k^2 - 11k - 40 = 0$

Question 8 continued

(b) ① Factorise $2k^2 - 11k - 40 = 0$

$$2k^2 - 11k - 40 = 0$$
$$(2k+5)(k-8) = 0$$

$2k + 5 = 0$ or $k - 8 = 0$

$k = \frac{-5}{2}$ or $k = 8$

② Find r using both values of k:

If $k = -5/2$: $r = \dfrac{-5/2 - 2}{-5/2 + 4} = -3$

If $k = 8$: $r = \dfrac{8-2}{8+4} = \dfrac{1}{2}$

③ Use sum to infinity formula:

$\hookrightarrow S_\infty$ converges ONLY if : $-1 < r < 1$

where $S_\infty = \dfrac{a}{1-r}$

$r = -3 \rightarrow$ reject as $-3 < -1$

$r = \frac{1}{2}$

$$S_\infty = \frac{8+4}{1-\frac{1}{2}} = \frac{12}{1-\frac{1}{2}} = 24$$

Question 8 continued

(Total for Question 8 is 13 marks)

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

t must be greater than 2 in any solution.

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

cubic function

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

(a) $3\log_2(t+4) - 2\log_2(t-2) = 7$

→ Substitute $t=4$ into equation:

$3\log_2(4+4) - 2\log_2(4-2) = 7$

$3\log_2 8 - 2\log_2 2 = 7$ → $\log_a a = 1$

$3(3) - 2(1) = 7$

$9 - 2 = 7$

$7 = 7$ ✓

(b) • Recall the rules of logarithms:

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$x\log_a(b) = \log_a(b)^x$

① rewrite coefficients as powers:

$3\log_2(t+4) - 2\log_2(t-2) = 7$

Question 9 continued

$$\log_2(t+4)^3 - \log_2(t-2)^2 = 7$$

② use the rules of logarithms

$$\log_2\left(\frac{(t+4)^3}{(t-2)^2}\right) = 7$$

③ Use the rule :

$\log_2 a = b \longrightarrow 2^b = a$

$$\log_2\left(\frac{(t+4)^3}{(t-2)^2}\right) = 7$$

so : $\dfrac{(t+4)^3}{(t-2)^2} = 2^7$

④ expand brackets :

$$\frac{(t+4)^2(t+4)}{(t-2)^2} = 128$$

$$\left[\frac{(t^2+8t+16)(t+4)}{t^2-4t+4} = 128\right] \times (t^2-4t+4)$$

$$t^3 + 4t^2 + 8t^2 + \boxed{32t} + \boxed{16t} + 64 = 128(t^2-4t+4)$$

$$t^3 + 12t^2 + 48t + 64 = 128t^2 - 512t + 512$$

$$t^3 + \underbrace{12t^2 - 128t^2} + \underbrace{48t + 512t} + \underbrace{64 - 512} = 0$$

$$t^3 - 116t^2 + 560t - 448 = 0$$

(c) ① $3\log_2(t+4) + 2\log_2(t-2) = 7$

is the same as :

Question 9 continued

(2) $t^3 - 116t^2 + 560t - 448 = 0$

So: If $t=4$ is a solution to (1), then it is also a solution to (2)

Since $f(4) = 0$, then $(t-4)$ is a factor of $t^3 - 116t^2 + 560t - 448$.

Use algebraic division to find quotient:

$$t-4 \overline{\big)\, t^3 - 116t^2 + 560t - 448}$$

(1) Divide first term of dividend by first term of divisor: $\frac{t^3}{t} = t^2 \rightarrow$ first term of quotient

$$\begin{array}{r|l} & t^2 \\ \hline t-4 & t^3 - 116t^2 + 560t - 448 \end{array}$$

$\div$

(2) Multiply out and write under:

$$\begin{array}{r|l} & t^2 \\ \hline t-4 & t^3 - 116t^2 + 560t - 448 \\ & t^3 - 4t^2 \\ \hline \end{array}$$

(3) Subtract then bring down next term:

$$\begin{array}{r|l} & t^2 \\ \hline t-4 & t^3 - 116t^2 + 560t - 448 \\ & -(t^3 - 4t^2) \\ \hline & -112t^2 + 560t \end{array}$$

(4) Divide first term of expression by first term of divisor: $\frac{-112t^2}{t} = -112t \rightarrow$ next term of quotient

$$\begin{array}{r|l} & t^2 - 112t \\ \hline t-4 & t^3 - 116t^2 + 560t - 448 \\ & -(t^3 - 4t^2) \\ \hline & -112t^2 + 560t \end{array}$$

Question 9 continued

(5) Multiply out and write under:

$$
\begin{array}{r|l}
 & t^2 - 112t \\
\hline
t - 4 & t^3 - 116t^2 + 560t - 448 \\
 & -(t^3 - 4t^2) \\
\hline
 & -112t^2 + 560t \\
 & -112t^2 + 448t \\
\hline
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & t^2 - 112t \\
\hline
t - 4 & t^3 - 116t^2 + 560t - 448 \\
 & -(t^3 - 4t^2) \\
\hline
 & -112t^2 + 560t \\
 & -(-112t^2 + 448t) \\
\hline
 & 112t - 448
\end{array}
$$

(7) Divide first term of expression by first term of divisor: $\frac{112t}{t} = 112 \rightarrow$ next term of quotient

$$
\begin{array}{r|l}
 & t^2 - 112t + 112 \\
\hline
t - 4 & t^3 - 116t^2 + 560t - 448 \\
 & -(t^3 - 4t^2) \\
\hline
 & -112t^2 + 560t \\
 & -(-112t^2 + 448t) \\
\hline
 & 112t - 448
\end{array}
$$

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & t^2 - 112t + 112 \\
\hline
t - 4 & t^3 - 116t^2 + 560t - 448 \\
 & -(t^3 - 4t^2) \\
\hline
 & -112t^2 + 560t \\
 & -(-112t^2 + 448t) \\
\hline
 & 112t - 448 \\
 & 112t - 448
\end{array}
$$

(Total for Question 9 is 9 marks)

(9) Subtract:

$$
\begin{array}{r|l}
 & t^2 - 112t + 112 \\
\hline
t - 4 & t^3 - 116t^2 + 560t - 448 \\
 & -(t^3 - 4t^2) \\
\hline
 & -112t^2 + 560t \\
 & -(-112t^2 + 448t) \\
\hline
 & 112t - 448 \\
 & -(112t - 448) \\
\hline
 & 0
\end{array}
$$

So $t^3 - 116t^2 + 560t - 448 = 0$

$= (t-4)(t^2 - 112t + 112) = 0$

FACTORISE: $1t^2 - 112t + 112 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-112) \pm \sqrt{(-112)^2 + 4(1)(112)}}{2(1)}$$

$$= \frac{112 \pm \sqrt{12\,096}}{2} = \frac{112 \pm 24\sqrt{21}}{2}$$

$= 56 + 12\sqrt{21}$ or $56 - 12\sqrt{21}$

≈ 111 $\quad$ ≈ 1.009 ✗

REJECT

$\log_2(1.009 - 2) = \log_2(-0.991)$

and log of a negative value is not possible

So: $t = 4,\ 56 + 12\sqrt{21}$

10.

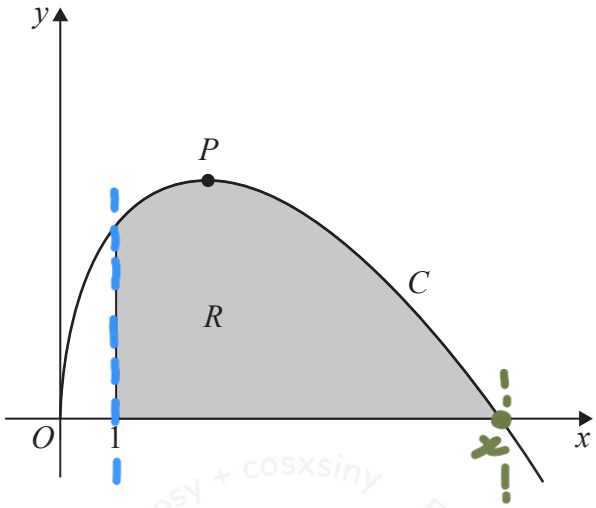


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C. $\frac{dy}{dx} = 0$

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

(a) $y = \frac{9x - x^2}{2\sqrt{x}}$

rewrite :

$y = \frac{9x}{2x^{1/2}} - \frac{x^2}{2x^{1/2}}$

$y = \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{3/2}$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{9}{2}\right)x^{\frac{1}{2}-1} - \frac{3}{2}\left(\frac{1}{2}\right)x^{\frac{3}{2}-1} = 0$

DIFFERENTIATION

$y = ax^n$

$\frac{dy}{dx} = nax^{n-1}$

Question 10 continued

$$\frac{dy}{dx} = \frac{9}{4}x^{-1/2} - \frac{3}{4}x^{1/2} = 0$$

$$\left(\frac{9}{4x^{1/2}} - \frac{3}{4}x^{1/2} = 0\right) \times x^{1/2}$$

$$\frac{9}{4} - \frac{3}{4}x = 0$$

$$\cancel{+}\frac{3}{\cancel{4}}x = \cancel{+}\frac{9}{\cancel{4}}$$

$$x = 3$$

(b) Area $R = \int_1^x \frac{9}{2}x^{1/2} - \frac{1}{2}x^{3/2}\,dx$

① Find upper bound x :

$$\frac{9x - x^2}{2\sqrt{x}} = 0$$

$$9x - x^2 = 0$$

$$x(9 - x) = 0$$

$x = 0$ or $x = 9$

reject

② Integrate : $\int_1^9 \frac{9}{2}x^{1/2} - \frac{1}{2}x^{3/2}\,dx$

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

Question 10 continued

$$\int_1^9 \frac{9}{2}x^{1/2} - \frac{1}{2}x^{3/2} \, dx$$

$$= \left[\frac{\cancel{2}}{3}\left(\frac{9}{\cancel{2}}\right)x^{3/2} - \frac{\cancel{2}}{5}\left(\frac{1}{\cancel{2}}\right)x^{5/2}\right]_1^9$$

$$= \left[3x^{3/2} - \frac{1}{5}x^{5/2}\right]_1^9$$

$$= \left[3(9)^{3/2} - \frac{1}{5}(9)^{5/2}\right] - \left[3(1)^{3/2} - \frac{1}{5}(3)^{5/2}\right]$$

Area R $= \frac{148}{5}$

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS